Application of Randomized Algorithm in Robust Eigenstructure Assignment

Titas Bera, M. S. Bhat and Debasish Ghose

***Abstract*— Several robust eigenstructure assignment methods for MIMO systems try to exploit non-uniqueness of controller matrix to optimize certain performance index. In general these methods lead to computational intractability or become too conservative design. Therefore the scope of using randomized algorithms as an alternative exists. Based on well established statistical learning theory algorithms this paper provides methodologies to design robust controller under structured uncertainty by eigenstructure assignment method. Also a Monte Carlo based computational procedure for a low sensitivity regional pole assignment problem is illustrated. With randomized methods, a controller can be easily found working satisfactorily for most of the time, while the price to pay for is its probabilistic completeness.**

I. INTRODUCTION

Eigenstructure assignment is a well established method that directly incorporates classical time domain design specifications. Initiated by Wonham [1] and later, Moore [2] described the non-uniqueness of the controller which can assign a self conjugate set of eigenvalues for MIMO system. The seminal paper of Srinathkumar [3] clearly describes the freedom available to the designer in selecting maximum number of eigenvalues and number of entries in each corresponding eigenvector. There exist several iterative and non-iterative methods which exploits this freedom to design appropriate controller for meeting the design specifications. Like, since the early design periods, attempts have been made to assign eigenvalues having low sensitivity with respect to perturbation in system matrices for better robustness. These can be thought of some what indirect approaches to design a robust controller. In these methods, the sensitivity measure considered in general, is the overall eigenvalue condition number which is a function of quadratic norm of modal matrix. There exists several methods for the problem of exact eigenvalue placement with minimum overall eigenvalue condition number. One can find in [4] details of low sensitivity eigenstructure assignment methods for exact eigenvalue location. In practice, it is hardly required to place the eigenvalue exactly. Instead, often the design requirement is to place the poles in pre-specified regions in the complex plane. A low sensitivity eigenvalue assignment within a specific region is considered by Lam and Tam [5]. Their approach is based on gradient based minimization technique to find the optimum pole location, which corresponds to a minimum Frobenius condition number of the closed loop state matrix. In general such a design is over conservative when compared to designs based on minimization of quadratic overall eigenvalue condition number. Note that quadratic condition number minimization with respect to a relaxed pole location is in general a non convex problem.

In contrast to minimizing overall eigenvalue sensitivity, several direct attempts have been made to ensure both time domain and frequency domain robust stability and performance measures. This is done by combining frequency domain stability and performance measures along with time domain design specification to realize joint optimum robustness. In [4] and [6], minimization of an infinity norm of mixed sensitivity measure over all possible controller is done using genetic algorithm and gradient based methods. But as shown by researchers, H_∞ norm minimization methods results in very conservative controller design.

From early 1990's control researchers began to consider the computational complexity of several control problems. One can find a survey of existing results regarding computational complexity of many robust control problems in [7]. In particular, simultaneous stabilization of linear systems with static state/output feedback when the controller parameters are constrained is proved to be NP-Hard. (Clearly one can relate the problem of simultaneous stabilization of linear systems by constrained controller to the problem of robust constrained eigenstructure assignment methods, since in the later case, the designer generally put constraints on some entries of desired eigenvector matrix to shape the time response/ mode decoupling). Details can be found in [8] and [9].

With such pessimistic result, an alternative approach that is recently gaining popularity is to use randomized algorithm to break the curse of dimensionality. In general these algorithm are not required to work in all cases, but rather most of the cases. These algorithms have polynomial time complexity. The price one pays is that, these algorithms are only probabilistically complete. Early attempt to bring randomization into control is initiated by Ray and Stengel [10]. More recently Khargonekar and Tikku [11], Tempo and Dabbane [12] and Vidyasagar [13] contributed to this field.

With this motivation, in this paper a randomized approach to low sensitivity eigenstructure assignment for relaxed pole location is developed. Also, application of randomized algorithm originated from statistical learning theory

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on eigenstructure assignment in designing robust control under structured/parametric uncertainty is shown. The paper is organized as follows: in Section II classical eigenstructure assignment method is reviewed. In Section III a robustness measure is defined and consequently general nature the objective functions is investigated. A randomized algorithm method to solve low sensitivity eigenstructure assignment problem is presented. Section IV contains a direct application of statistical learning theory to the design of constrained robust controllers against parametric uncertainty, while in subsequent section we discuss the result and conclude.

II. Eigenstructure Assignment Method for LTI Systems

Consider a MIMO linear time invariant system described by

$$\dot{x} = Ax + Bu \quad (1)$$
$$y = Cx \quad (2)$$

where the state vector $x \in \mathbb{R}^n$, control input vector $u \in \mathbb{R}^m$, output vector $y \in \mathbb{R}^r$ and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{r\times n}$ system matrices. Assume $rank(B) = m$ and $rank(C) = r$ (This full rank assumption is not strictly required). Define a required closed-loop self conjugate eigenvalue set $\wedge = \{\lambda_i : \lambda_i \in \mathbb{C}, i = 1,2,\dots n\}$. For simplicity assume all required eigenvalues are distinct. The linear output feedback law is applied to the open loop system as $u = Ky$ where $K \in \mathbb{R}^{m\times r}$ such that the closed loop eigenvalue specified in $\wedge$ is attained. Let us denote the i^{th} left and right eigenvector corresponding to λ_i as L_i and R_i respectively. The modal and spectral matrices defined as

$$R = [R_1 \;\; R_2 \;\; \dots R_n] \in \mathbb{R}^{n\times n} \quad (3)$$
$$L = [L_1 \;\; L_2 \;\; \dots L_n] \in \mathbb{R}^{n\times n} \quad (4)$$

and $D_\wedge = diag[\lambda_1, \lambda_2, \dots \lambda_n] \in \mathbb{R}^{n\times n}$.
Then the forced response of the system is given by

$$y(t) = CR[e^{D_\wedge t}L^T x(0) + \int_0^t e^{D_\wedge (t-\tau)}L^T Br(\tau)d\tau] \quad (5)$$

Eigenvalues, together with eigenvectors plays an important role in shaping response. Judicial choice of eigenvector can prevent a mode from being excited by input. Using the control law $u = ky$, $\max(m,r)$ eigenvalues may be assigned and $\min(m,r)$ entries of each corresponding eigenvector can be chosen precisely [3]. Assume for simplicity that the set of open and closed loop eigenvalues have no element in common), that is $|\lambda_i I - A| \neq 0$ for $i = 1,2,\dots n$ then

$$R_i = (\lambda_i I - A)^{-1}BKCR_i = (\lambda_i I - A)^{-1}BW_i \quad (6)$$

where $W_i = KCR_i$ Eqn (6) can be written in matrix form

$$[W_1 \;\; W_2 \;\; \dots W_n] = KC[R_1 \;\; R_2 \;\; \dots R_n] \quad (7)$$
$$K = W(CR)^{-1} \quad (8)$$

The determination of controller matrix K therefore is straightforward. Note that the accuracy of computing K is actually depends on condition number of right eigenvector matrix.

III. Low Sensitivity Eigenstructure Assignment under Regional Pole Constraints

Define $\wedge = \{\lambda_1, \lambda_2, \dots \lambda_n\}$ to be a self conjugate set of eigenvalues. Now given a system matrix (1) one can always select a $K = WR^{-1}$ such that $\wedge$ is attainable. Define $J(R) = \eta(R) = \|R\|_2\|R^{-1}\|_2$ as the quadratic condition number. The control problem then is to select K such that $J(R)$ is minimized. That is,

$$\hat{K} = \arg\min_K J(R) \quad (9)$$

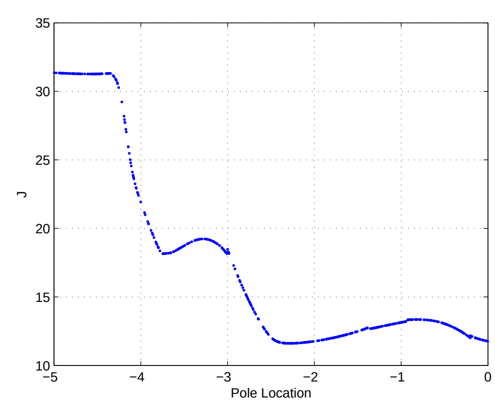
Let us define $\hat{\wedge} = \{\lambda_i \in \Omega_i, i = 1,2,\dots n\}$ that is, instead of specifying an exact location for eigenvalues, the requirement is now to place the poles in a region defined as an union of disconnected closed regions on the complex plane. Assume each $\Omega_i, i = 1,2,\dots,n$ are convex. Then the problem is to find the most optimum pole location and consequently the controller that yields minimum quadratic condition number of the closed loop modal matrix. i.e.

$$\hat{\wedge}_{\text{opt}} = \arg\min_{\hat{\wedge}} \max_{K_{\hat{\wedge}}} \frac{1}{J(R)} \quad (10)$$

one can think of gradient based minimization techniques to find $\hat{\wedge}_{\text{opt}}$, but in general the cost function, can be very arbitrary non-convex with respect to closed loop pole locations.

This non-convex nature is illustrated in the subsequent example.
Numerical Example 1: Consider a LTI system described as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1.8900 & 0.3900 & -5.5550 \\ 0 & -0.0340 & -2.9800 & 2.4300 \\ 0.0350 & -0.0011 & -0.9900 & -0.2100 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 \\ 0.7600 & -1.600 \\ -0.9500 & -0.0320 \\ 0.0030 & 0 \end{bmatrix}$$



The desired closed loop pole locations are $\lambda_1 = -4.5$,$\lambda_2 = -6.5$, $\lambda_3 \in [-5,0]$, and $\lambda_4 = -3$. The cost function $J(R,\lambda) = \|R\|_2\|R^{-1}\|_2$ where R is the closed loop modal

matrix. To construct an approximate nature of cost function variation with λ_3, we took 500 samples of λ_3 variation. In each case we use the Rank-n Update eigenstructure assignment method [16] to find the minimum value of $J(R^*, \lambda_3 = \lambda_{3i}, i = 1, 2, ...500)$, and each case the method has converged giving best eigenstructure for that choice of λ_3. The figure clearly showing a non convex nature of the cost function. In general the nature of the cost function can be very arbitrary and gradient based minimization approach may lead to local minima. One way to avoid the problem is to start gradient based minimization with several initial condition, but in general these are difficult to estimate a priori. Another viable approach is to use Monte Carlo technique for finding probably approximate minima. In Monte Carlo procedure, instead of finding true J^* one tries to find an estimate of J^* with an accuracy ε and confidence $1-\delta$. From classical one sided Chernoff bound one can find the minimum number of i.i.d samples to estimate the quantity J^* with confidence $1-\delta$ and accuracy ε. More specifically if $M \geq \frac{1}{2\varepsilon^2} \ln \frac{1}{\delta}$ then $Pr\{|\hat{J}^* - J^*| \leq \varepsilon\} \geq 1-\delta$. Now, we present a very simple Monte Carlo (non adaptive random search method) based procedure for low sensitivity eigenstructure assignment.

Algorithm: Low Sensitivity Eigenstructure Assignment
Generate M i.i.d self conjugate pole location set $S(\lambda_i)$ according to a probability distribution (P_x)

For $i \leftarrow 1$ to M do
calculate min η and K for $S(\lambda_i)$ using any method
$\triangleright$ One can use Direct method/ Rank N update/ Rank
$\triangleright$ one Update/ KNV method etc.
Return The empirical optimum pole locations
$\hat{S}(\lambda_i) = \arg\min_i \eta^*(S(\lambda_i))$

Below is an example of low sensitivity state feedback eigenstructure assignment using Monte carlo method.
Numerical Example 2: Consider the previous LTI model defined in numerical example 1. The system is controllable. We designed a low sensitivity state feedback eigenstructure assignment for the following closed loop pole specifications. The closed loop pole location are specified as $\lambda_1 \in \{(-3.\underline{5}+ R\cos\theta) + j(3 + R\sin\theta), R \in [0,], \theta \in [0,-60]\}$, $\lambda_2 = \bar{\lambda}_1$, $\lambda_3 \in [-1.6, -0.8]$ and $\lambda_4 = -3$. For an accuracy of 0.01 and confidence level of 0.99 we need to generate at least $23,025$ samples to estimate the value of $\hat{J}^*$. We used a Rank-n-Update method to find the eigenvector corresponds to minimum quadratic condition number for a given eigenvalue. In this case, for every sampled eigenvalue in the specified region the Rank n Update algorithm converged. Finally we get $\eta(R) = 3.6083$. For simplicity we have not assume any structure of the eigenvector matrix. The corresponding controller

$$K = \begin{bmatrix} 0.2188 & 0.0734 & -0.7441 & -0.5639 \\ -5.2205 & -0.7562 & -0.6763 & 3.3718 \end{bmatrix}$$

and the eigenvalue locations are

$$\begin{aligned} \lambda_1 &= -1.5783 + 2.4560i \\ \lambda_2 &= -1.5783 - 2.4560i \\ \lambda_3 &= -0.9158 \\ \lambda_4 &= -3.0000 \end{aligned}$$

IV. Robust Control Design using Eigenstructure Assignment

Modelling errors are inevitably introduced when a model is obtained by approximating the system dynamics. These errors can be introduced in the form of structured / parametric or unstructured uncertainty. Due to these uncertainties, the eigenvalue of nominal system and the true system may differ, resulting in a difference in response characteristics. Low sensitivity eigenstructure assignment is an indirect method to tackle the effect of uncertainty but as shown in [17] that a small improvement in the sensitivity measure may result in a significant degradation in frequency domain measure of robustness. In contrast to the low sensitivity eigenstructure assignment, robust control design by H_∞ optimization methods try to find a feedback controller that guarantees closed loop stability and performance robustness under all uncertainties. But the price to pay for this is, it often compromises with system bandwidth, as shown in [22], resulting sluggish response characteristics. Since time domain response characteristics governed by system eigenstructure, are equally important design concern, therefore it is useful to opt for a design method which satisfies both the time domain performance specifications provided by eigenstructure and frequency domain robust stability and performance specification. For MIMO systems, for a specific pole assignment problem, resulting controller is non unique. Therefore one can select a controller gain K which optimizes a Robust performance index. Early contribution on this area was by [6] and [18]. These works are concerned with optimization of a mixed sensitivity performance index as robustness measure under unstructured uncertainty. In the present work we tried to investigate the same for structured uncertainty.

First consider the problem of robust stability using eigenstructure assignment under structured uncertainty. Earlier, Sobel et al [19] tried to solve this problem by placing the closed loop eigenvalues far away form the $j\omega$ axis.

Although, only eigenvalue assignment can guarantee robust stability, but it alone cannot guarantee robust performance. Moreover the robust stability result is very conservative and not many design freedom exists. Therefore it is worth to look at the design freedom offered by simultaneous eigenvalue and eigenvector assignment. Let us formulate the problem as to ensure stability and performance robustness under all perturbation by designing suitable eigenstructure. Denote uncertainty affecting the LTI system described as $\mathbb{D} = \{\delta \in F : \delta = bdiag(q_1 I_1, q_2 I_2, ...q_m I_m, \Delta_1, ...\Delta_b)\}$ and $B_D = \{\delta \in \mathbb{D} : \|q\|_p \leq \rho, \bar{\sigma}(\Delta_i) \leq \rho, i = 1, 2, ...b\}$. We consider the family of uncertain systems obtained when the uncertainty

δ varies over $B_D(\rho)$ and we say that a certain property, e.g. stability or performance is robustly satisfied if it satisfied for all members of the family. The problem becomes: For a plant with such structured/parametric uncertainty design a controller using eigenstructure assignment method so that the following robust stability and performance requirements are satisfied for all members in the family of uncertain plants.

1) for all λ_p, $Re(\lambda_p) < 0$ i.e. perturbed plant will be robustly stable
2) $\sup \bar{\sigma} \begin{bmatrix} S \\ T \end{bmatrix} < \varepsilon$ where $S(s)$ and $T(s)$ are the sensitivity and complementary sensitivity transfer function.

During recent years it has been reported that several problem in robustness analysis and synthesis are either NP complete or NP hard. A survey of NP hard problems in control theory can be found in the work of Blondel and Tsitkilis [7], Nemirovskii [20], Poljak and Rohn [21]. These results shows that various problems in robust control are practically unsolvable if the number of variables become sufficiently large. The problem of checking robust stability and performance under structured/ parametric uncertainty is proven as NP-Hard. However, parametric uncertainty can also be handled in H_∞ framework, but then the design will be too conservative. μ Analysis/Synthesis approach although can handle parametric uncertainty, but computational complexity of μ calculation is again NP Hard. Also stability evaluation of interval matrices is itself a NP hard problem. To overcome the intractability issues and conservatives, recent trends are to consider the problem of stability and performance robustness in probabilistic framework [24]. Define each triple (A,B,C) where (A,B,C) are system matrices, belongs to $A \times B \times C$. Denote $A \times B \times C$ as S. Now suppose P is a probability measure on the set S. Define ε be a given accuracy parameter. For each matrix $K \in \mathbb{K}$ define $\mathbb{T}(K) = \{(A,B,C) : J_i(A,B,C) \leq \gamma_i, i = 1,2...n\}$. In probabilistic framework the question is: Does there exist a matrix $K \in \mathbb{K}$ such that $P(\mathbb{T}(K)) \geq 1-\varepsilon$? In other words, instead of looking for a matrix $K \in \mathbb{K}$ which satisfies worst case performance, one compromises the search for a $K \in \mathbb{K}$ satisfies the performance index for most of the cases except possibly those belonging to set of volume no larger than ε.

Now we describe one important notion of probable approximate minima as found in [14].

Definition: Suppose $f : Y \to \mathbb{R}$, that P_Y is a given probability measure on Y and that $\varepsilon, \alpha > 0$ are given numbers. A number $f_0 \in \mathbb{R}$ is said to be a near minimum of $f(.)$ to accuracy ε and level α, or a probably approximate minimum of $f(.)$ to accuracy ε and level α if $f_0 \geq f^* - \varepsilon$ and in addition $P_Y\{y \in Y : f(y) < f_0 - \varepsilon\} \leq \alpha$.

Based on this definition sample complexity bound has been established in [14], and two algorithms have been proposed. The first one is universal while the second one is applicable to situations where associated family of functions has the uniform convergence of empirical mean (UCEM)property. Now, a collection of sets can have UCEM property only when its VC dimension [14] is finite. For the class of function $\mathbb{T}(K)$. In [13] it has been established that they have finite VC dimension and thereby poses UCEM property. However sample complexity bounds in this case are two large to be useful. Hence, we choose a more general class of algorithm to design a probabilistic eigenstructure assignment.

Algorithm: Probabilistic Eigenstructure Assignment
Choose integers $M \geq \frac{1}{\log \frac{1}{1-\alpha}} \ln \frac{2}{\delta}$ and $N \geq M \frac{1}{2\varepsilon^2} \ln \frac{4N}{\delta}$ Generate N i.i.d samples $x_1, x_2, ...x_N$ from set S according to (P_x) and M i.i.d samples $k_1, k_2, ...k_m$ from $\mathbb{K}$ according to P_k

For $j \leftarrow 1$ to M do
calculate $\hat{f}_i = \frac{1}{M} \sum_{i=1}^{M} T(k_j, x_i)$
Return The empirical optimum controller
$\hat{K}(k_j) = \arg\min_j \hat{f}_i$

Then with confidence $1-\delta$, it can be said that $\hat{f}_0$ is a probably approximate near minimum of $f(.)$ to accuracy ε and level α.

Numerical Example 3: Consider a LTI plant model described as

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 2 \\ -2 & -2 & -3 \end{bmatrix} B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 0 \end{bmatrix} C = I_{3\times 3}$$

The perturbation model considered as $\Delta = diag\{\delta_{11}, \delta_{22}, \delta_{33}\}$ where $\delta_{11} \in [-1,0]$,$\delta_{22} \in [-1,0]$,$\delta_{33} \in [-1,0]$. The controller has the structure $K = WR^{-1}$ where R is the right eigenvector matrix and W is the design vector matrix. Let's assume we have right eigenvector structure constrained as

$$R = \begin{bmatrix} \times & 1 & 0 \\ 1 & \times & 0 \\ 0 & 1 & \times \end{bmatrix}$$

where a $\times$ signifies arbitrary values are possible. Therefore we define $K = K(y_1, y_2, y_3)$ where y_1, y_2, y_3 are the diagonal elements of right eigenvector matrix. Define cost function as

$$J = \left\{ \begin{array}{cc} 1 & \text{if}(A(x), B, K, C)\text{unstable} \\ \frac{\left\|(I-GK)^{-1}\right\|_\infty}{1+\left\|(I-GK)^{-1}\right\|_\infty} & \text{otherwise} \end{array} \right\}$$

selecting $\varepsilon = 0.01, \alpha = 0.01, \delta = 0.05$ results in $M \geq 367$ and $N \geq 22339$. Desired pole locations are $-1, -2, -5$. After running the algorithm the controller matrix found as,

$$K = \begin{bmatrix} -3.0566 & -0.1954 & 1.2314 \\ 3.1167 & 3.5937 & 1.5775 \end{bmatrix}$$

The right eigenvector matrix

$$R = \begin{bmatrix} -0.2336 & 0.5461 & 0.0927 \\ 0.1701 & -0.5118 & 0.0556 \\ -0.2766 & 0.8865 & 0.6306 \end{bmatrix}$$

and corresponding $\|S(s)\|_\infty = 3.54$. Note that this controller does not stabilizes the plant under all uncertainties. As a counter example, if the perturbed matrix is

$$A(\Delta) = \begin{bmatrix} -0.9002 & 2 & 0 \\ 0 & -1.8709 & 2 \\ -2 & -2 & -3.4576 \end{bmatrix}$$

then the closed loop system has eigenvalue $\{0.033, -3.76, -5.5\}$.

V. Conclusion

We have shown how randomized algorithm can be used in Low sensitivity eigenstructure assignment for relaxed pole locations and robust control design using eigenstructure assignment method under structured perturbation. The algorithms are computationally straightforward to implement, although the estimate of sample complexity is conservative. Numerical examples are shown for state feedback eigenstructure assignment although these procedure can be extended for output feedback.

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